Complementary Material: Parametric Optimization of EN-31 Steel Using Electric Discharge Machining

Material Complementario: Optimización paramétrica del acero EN-31 mediante mecanizado por descarga eléctrica

Material Complementar: Otimização paramétrica do aço EN-31 utilizando usinagem por eletroerosão

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Summary. - This investigative study was conducted for the parametric optimization of EN-31 material by using a non-conventional machining known as Electric discharge machining (EDM). EN-31 is a steel alloy that is generally used in aerospace industry, automotive parts manufacturing, die making etc. because it possesses high degree of rigidity with extremely good compressive strength and resistance to abrasion. The primary objective of this study was to analyze the impact of four input factors i.e. pulse on time (Ton), pulse off time (Toff), current (LV), voltage (HV) on the five output responses i.e. machining time (Tm), MRR, EWR, Ra and base radius (R). In this study design of experiment (DOE) approach with full factorial design was systematically conducted. Basic experimental runs were prepared and performed and after that data was analyzed using ANOVA to identify significant input factors that has most impact on each output response that are mentioned above. After identification of significant factors optimized input factors and output responses were calculated using ANOVA. The results showed that for machining time (Tm), LV and Ton were significant factors with optimized values of 50 A and 6.5 µs, respectively, resulting in a Tm of 654.29 seconds. For material removal rate (MRR), Ton was the significant factor with an optimized value of 6.5 µs, achieving a maximum MRR of 0.0157 g/min. For electrode wear rate (EWR), LV and Ton were significant with optimized values of 30 A and 4 µs, respectively, resulting in a minimum EWR of 0.07 g/min. Ra optimization revealed that the combination of HV, LV, Ton and Toff were significant, with optimized settings of 50 A, 0.7 V, 4.0 µs and 6.5 µs, respectively, yielding a minimum Ra of 0.018 mm. For base radius (R), the significant factors were HV, LV, Ton and Toff, with optimized values of 0.6152 V, 50 A, 6.5 µs and 6.5 µs, respectively, resulting in a base radius of 1.5 mm. This parametric optimization is extremely significant because EN-31 is a material used in critical applications requiring high strength, hardness and abrasion resistance such as automobile engine components, aerospace rocket parts and dies subjected to extreme temperatures and pressures throughout their lifecycle. Optimizing EDM parameters facilitates the use of this non-conventional machining process for manufacturing EN-31 parts thus enabling researchers, manufacturers, designers and industry practitioners to achieve higher productivity, excellent surface finishes and lower manufacturing costs as compared to traditional manufacturing techniques. This optimization allows for more efficient and effective production of high-performance parts thus making it an invaluable advancement in various industrial sectors.

Keywords: *EDM; Parameters; Machining; Processing; Roughness, EN-31; Optimization; DOE.*

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Resumen. - *Este estudio de investigación se realizó para la optimización paramétrica del material EN-31 utilizando un mecanizado no convencional conocido como mecanizado por descarga eléctrica (EDM). El EN-31 es una aleación de acero que se utiliza generalmente en la industria aeroespacial, la fabricación de piezas de automóviles, la fabricación de matrices, etc., debido a que posee un alto grado de rigidez con una resistencia a la compresión y a la abrasión extremadamente buenas. El objetivo principal de este estudio fue analizar el impacto de cuatro factores de entrada, a saber, tiempo de pulso encendido (Ton), tiempo de pulso apagado (Toff), corriente (LV) y voltaje (HV) en las cinco respuestas de salida, a saber, tiempo de mecanizado (Tm), MRR, EWR, Ra y radio base (R). En este estudio se llevó a cabo sistemáticamente un enfoque de diseño de experimentos (DOE) con un diseño factorial completo. Se prepararon y realizaron ensayos experimentales básicos y luego se analizaron los datos utilizando ANOVA para identificar los factores de entrada significativos que tienen el mayor impacto en cada respuesta de salida mencionada anteriormente. Después de la identificación de los factores significativos, los factores de entrada optimizados y las respuestas de salida se calcularon utilizando ANOVA. Los resultados mostraron que para el tiempo de mecanizado (Tm), LV y Ton fueron factores significativos con valores optimizados de 50 A y 6,5 µs, respectivamente, lo que resultó en un Tm de 654,29 segundos. Para la tasa de remoción de material (MRR), Ton fue el factor significativo con un valor optimizado de 6,5 µs, logrando un MRR máximo de 0,0157 g/min. Para la tasa de desgaste del electrodo (EWR), LV y Ton fueron significativos con valores optimizados de 30 A y 4 µs, respectivamente, lo que resultó en un EWR mínimo de 0,07 g/min. La optimización de Ra reveló que la combinación de HV, LV, Ton y Toff fueron significativas, con configuraciones optimizadas de 50 A, 0,7 V, 4,0 µs y 6,5 µs, respectivamente, lo que produjo un Ra mínimo de 0,018 mm. Para el radio base (R), los factores significativos fueron HV, LV, Ton y Toff, con valores optimizados de 0,6152 V, 50 A, 6,5 µs y 6,5 µs, respectivamente, lo que resultó en un radio base de 1,5 mm.*

Palabras clave: *EDM; Parámetros; Mecanizado; Procesamiento; Rugosidad; EN-31; Optimización; DOE.*

Resumo. - *Este estudo investigativo foi conduzido para a otimização paramétrica do material EN-31 utilizando uma usinagem não convencional conhecida como eletroerosão (EDM). O EN-31 é uma liga de aço geralmente utilizada nas indústrias aeroespacial, de autopeças, de matrizes, etc., devido à sua alta rigidez, excelente resistência à compressão e à abrasão. O objetivo principal deste estudo foi analisar o impacto de quatro fatores de entrada, ou seja, tempo de pulso ligado (Ton), tempo de pulso desligado (Toff), corrente (LV) e tensão (HV), sobre as cinco respostas de saída, ou seja, tempo de usinagem (Tm), taxa de remoção de material (MRR), taxa de desgaste da ferramenta (EWR), rugosidade superficial (Ra) e raio da base (R). Neste estudo, foi aplicada uma abordagem de planejamento de experimentos (DOE) com planejamento fatorial completo. Ensaios experimentais básicos foram preparados e realizados, e os dados foram analisados por meio de ANOVA para identificar os fatores de entrada significativos que mais impactam cada resposta de saída mencionada. Após a identificação dos fatores significativos, os fatores de entrada otimizados e as respostas de saída foram calculados utilizando ANOVA. Os resultados mostraram que, para o tempo de usinagem (Tm), a tensão de limiar (LV) e a taxa de variação da corrente (Ton) foram fatores significativos, com valores otimizados de 50 A e 6,5 µs, respectivamente, resultando em um Tm de 654,29 segundos. Para a taxa de remoção de material (MRR), a Ton foi o fator significativo, com um valor otimizado de 6,5 µs, atingindo uma MRR máxima de 0,0157 g/min. Para a taxa de desgaste do eletrodo (EWR), a LV e a Ton foram significativas, com valores otimizados de 30 A e 4 µs, respectivamente, resultando em uma EWR mínima de 0,07 g/min. A otimização da rugosidade média (Ra) revelou que a combinação de alta tensão (HV), LV, Ton e Toff foi significativa, com configurações otimizadas de 50 A, 0,7 V, 4,0 µs e 6,5 µs, respectivamente, resultando em uma Ra mínima de 0,018 mm. Para o raio da base (R), os fatores significativos foram HV, LV, Ton e Toff, com valores otimizados de 0,6152 V, 50 A, 6,5 µs e 6,5 µs, respectivamente, resultando em um raio da base de 1,5 mm.*

Palavras-chave: *EDM; Parâmetros; Usinagem; Processamento; Rugosidade; EN-31; Otimização; CORÇA.*

1. Experimental Inputs for EN-31 and all Replicates. -

Run Order	HV	LV	T_{on}	T_{off}	Work Piece Material	Electrode Material	Replicate Number	E_a	W_a	Duty Factor %
1	0.3	30	4	5.5	EN31	Copper	1	17.353	218.795	42%
2	0.7	30	4	5.5	EN31	Copper	1	17.22	218.632	42%
3	0.3	50	4	5.5	EN31	Copper	1	17.512	218.462	42%
4	0.7	50	4	5.5	EN31	Copper	1	17.379	218.297	42%
5	0.3	30	6.5	5.5	EN31	Copper	1	14.833	218.122	54%
6	0.7	30	6.5	5.5	EN31	Copper	1	14.709	217.94	54%
7	0.3	50	6.5	5.5	EN31	Copper	1	18.872	217.754	54%
8	0.7	50	6.5	5.5	EN31	Copper	1	18.75	217.578	54%
9	0.3	30	4	6.5	EN31	Copper	1	11.438	217.384	38%
10	0.7	30	4	6.5	EN31	Copper	1	11.31	217.22	38%
11	0.3	50	4	6.5	EN31	Copper	1	13.924	217.065	38%
12	0.7	50	4	6.5	EN31	Copper	1	13.774	216.896	38%
13	0.3	30	6.5	6.5	EN31	Copper	1	17.868	216.732	50%
14	0.7	30	6.5	6.5	EN31	Copper	1	17.75	216.558	50%
15	0.3	50	6.5	6.5	EN31	Copper	1	16.041	216.377	50%
16	0.7	50	6.5	6.5	EN31	Copper	1	15.901	216.182	50%
17	0.3	30	4	5.5	EN31	Copper	2	14.624	216.047	42%
18	0.7	30	4	5.5	EN31	Copper	2	14.515	215.907	42%
19	0.3	50	4	5.5	EN31	Copper	2	11.844	215.739	42%
20	0.7	50	4	5.5	EN31	Copper	2	11.719	215.561	42%
21	0.3	30	6.5	5.5	EN31	Copper	2	17.35	215.387	54%
22	0.7	30	6.5	5.5	EN31	Copper	2	17.23	215.201	54%
23	0.3	50	6.5	5.5	EN31	Copper	2	17.395	215.03	54%
24	0.7	50	6.5	5.5	EN31	Copper	2	17.268	214.838	54%
25	0.3	30	4	6.5	EN31	Copper	2	14.872	214.648	38%
26	0.7	30	4	6.5	EN31	Copper	2	14.735	214.492	38%
27	0.3	50	4	6.5	EN31	Copper	2	10.39	214.322	38%
28	0.7	50	4	6.5	EN31	Copper	2	10.252	214.155	38%
29	0.3	30	6.5	6.5	EN31	Copper	2	10.011	213.997	50%
30	0.7	30	6.5	6.5	EN31	Copper	2	9.887	213.82	50%
31	0.3	50	6.5	6.5	EN31	Copper	2	17.405	213.668	50%
32	0.7	50	6.5	6.5	EN31	Copper	2	17.266	213.493	50%
33	0.3	30	4	5.5	EN31	Copper	3	17.542	218.109	42%
34	0.7	30	4	5.5	EN31	Copper	3	17.418	217.955	42%
35	0.3	50	4	5.5	EN31	Copper	3	14.917	217.797	42%
36	0.7	50	4	5.5	EN31	Copper	3	14.785	217.603	42%
37	0.3	30	6.5	5.5	EN31	Copper	3	15.222	217.43	54%
38	0.7	30	6.5	5.5	EN31	Copper	3	15.088	217.256	54%
39	0.3	50	6.5	5.5	EN31	Copper	3	17.929	217.065	54%
40	0.7	50	6.5	5.5	EN31	Copper	3	17.807	216.91	54%
41	0.3	30	4	6.5	EN31	Copper	3	10.625	216.73	38%

42	0.7	30	4	6.5	EN31	Copper	3	10.49	216.57	38%
43	0.3	50	4	6.5	EN31	Copper	3	14.119	216.39	38%
44	0.7	50	4	6.5	EN31	Copper	3	13.97	216.226	38%
45	0.3	30	6.5	6.5	EN31	Copper	3	13.812	216.06	50%
46	0.7	30	6.5	6.5	EN31	Copper	3	13.672	215.903	50%
47	0.3	50	6.5	6.5	EN31	Copper	3	14.06	215.707	50%
48	0.7	50	6.5	6.5	EN31	Copper	3	13.942	215.538	50%

Table II. Experimental Inputs for EN-31 and all Replicates.

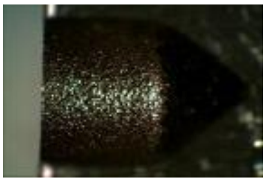
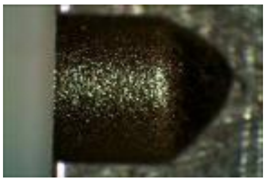
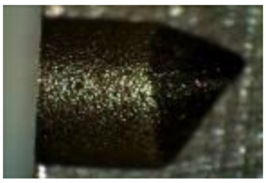
2. Experimental Responses for EN-31 material and all Replicates. -

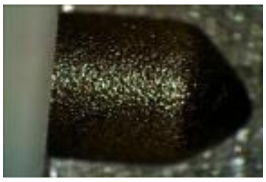
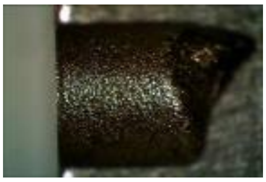
Run Order	Machining Time (sec)	MRR (g/min)	MRR (mm^3/min)	EW (g/min)	EW (mm^3/min)	Base Radius	Surface Roughness (R_a)
1	752	0.01301	1.67	0.01061	1.37	1.521	0.033
2	677	0.01507	1.93	0.01064	1.37	1.606	0.036
3	740	0.01338	1.71	0.01078	1.39	1.562	0.037
4	773	0.01358	1.74	0.01017	1.31	1.552	0.047
5	889	0.01228	1.57	0.00837	1.08	1.531	0.071
6	612	0.01824	2.33	0.01108	1.43	1.54	0.024
7	632	0.01671	2.14	0.01158	1.49	1.547	0.033
8	436	0.0267	3.42	0.01651	2.13	1.569	0.018
9	778	0.01265	1.62	0.00987	1.27	1.529	0.02
10	1003	0.00927	1.19	0.00754	0.97	1.527	0.021
11	1066	0.00951	1.22	0.00844	1.09	1.589	0.014
12	1104	0.00891	1.14	0.0075	0.97	1.531	0.022
13	617	0.01692	2.17	0.01147	1.48	1.539	0.047
14	427	0.02543	3.26	0.01728	2.23	1.573	0.022
15	638	0.01834	2.35	0.01317	1.7	1.615	0.019
16	452	0.01792	2.29	0.015	1.93	1.561	0.031
17	1251	0.00671	0.86	0.00523	0.67	1.723	0.013
18	1294	0.00779	1	0.00603	0.78	1.411	0.015
19	809	0.0132	1.69	0.00927	1.19	1.766	0.029
20	784	0.01332	1.71	0.01026	1.32	1.632	0.009
21	1051	0.01062	1.36	0.00685	0.88	1.667	0.012
22	535	0.01918	2.46	0.01234	1.59	1.57	0.028
23	458	0.02515	3.22	0.01664	2.14	1.6	0.021
24	635	0.01795	2.3	0.01162	1.5	1.592	0.045
25	1282	0.0073	0.93	0.00641	0.83	1.631	0.059
26	1357	0.00752	0.96	0.00557	0.72	1.493	0.035
27	1212	0.00827	1.06	0.00683	0.88	1.621	0.046
28	1107	0.00856	1.1	0.00748	0.96	1.612	0.018
29	931	0.01141	1.46	0.00799	1.03	1.588	0.024
30	1027	0.00888	1.14	0.00789	1.02	1.568	0.032
31	780	0.01346	1.72	0.01069	1.38	1.616	0.023
32	699	0.01485	1.9	0.01047	1.35	1.421	0.02

33	1073	0.00861	1.1	0.00693	0.89	1.833	0.011
34	1137	0.00834	1.07	0.0066	0.85	1.701	0.074
35	803	0.0145	1.86	0.00986	1.27	1.501	0.029
36	1260	0.00824	1.05	0.00595	0.77	1.458	0.04
37	1029	0.01015	1.3	0.00781	1.01	1.609	0.011
38	1033	0.01109	1.42	0.00697	0.9	1.587	0.03
39	837	0.01111	1.42	0.00875	1.13	1.63	0.006
40	810	0.01333	1.71	0.00956	1.23	1.587	0.03
41	1347	0.00713	0.91	0.00601	0.77	1.789	0.016
42	1214	0.0089	1.14	0.00618	0.8	1.697	0.017
43	902	0.01091	1.4	0.00945	1.22	1.412	0.027
44	1010	0.00986	1.26	0.00737	0.95	1.756	0.014
45	1014	0.00929	1.19	0.00828	1.07	1.464	0.029
46	688	0.01709	2.19	0.01099	1.42	1.86	0.014
47	552	0.01837	2.35	0.01283	1.65	1.777	0.02
48	787	0.01281	1.64	0.01067	1.37	1.23	0.021

Table III. Experimental Responses for EN-31 material and all Replicates.

3. EN-31 Workpiece Surface. - The images of EN-31 workpiece were taken at a magnification of 22 X.

Experimental Runs	**Replicate # 1**	**Replicate # 2**	**Replicate # 3**
1			
T_{on} = 4µs,T_{off}5.5 µs,Duty Factor = 42%, HV = 0.3, LV = 30			
2			
T_{on} = 4µs,T_{off}5.5 µs,Duty Factor = 42%, HV = 0.7, LV = 30			
3			
T_{on} = 4µs,T_{off}5.5 µs,Duty Factor = 42%, HV = 0.3, LV = 50			
4			
T_{on} = 4µs,T_{off}5.5 µs,Duty Factor = 42%, HV = 0.7, LV = 50			

Experimental Runs	**Replicate # 1**	**Replicate # 2**	**Replicate # 3**
5			
T_{on} = 6.5µs,T_{off}5.5 µs,Duty Factor = 54%, HV = 0.3, LV = 30			
6			
T_{on} = 6.5µs,T_{off}5.5 µs,Duty Factor = 54%, HV = 0.7, LV = 30			
7			
T_{on} = 6.5µs,T_{off}5.5 µs,Duty Factor = 54%, HV = 0.3, LV = 50			
8			
T_{on} = 6.5µs,T_{off}5.5 µs,Duty Factor = 54%, HV = 0.7, LV = 50			
9			
T_{on} = 4µs,T_{off}6.5 µs,Duty Factor = 38%, HV = 0.3, LV = 30			
10			
T_{on} = 4µs,T_{off}6.5 µs,Duty Factor = 38%, HV = 0.7, LV = 30			
11			
T_{on} = 4µs,T_{off}6.5 µs,Duty Factor = 38%, HV = 0.3, LV = 50			

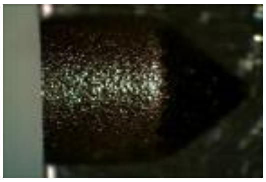
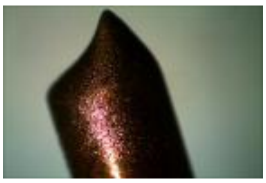
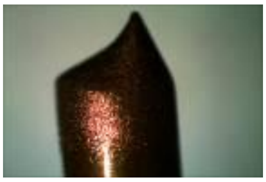
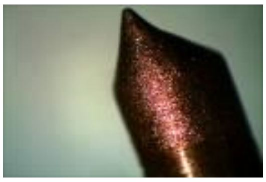
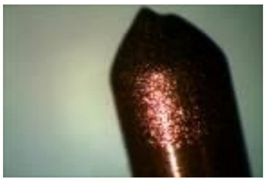
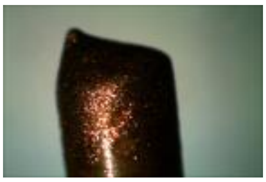
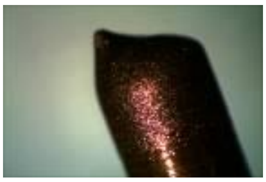
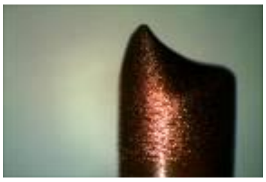
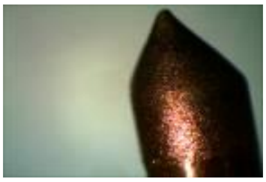
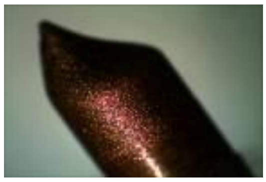
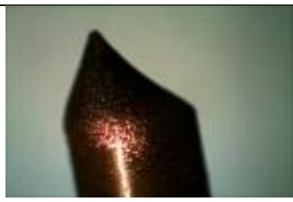
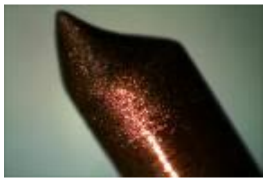
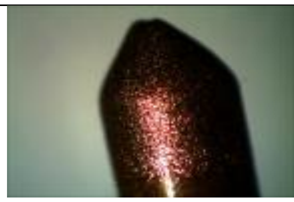
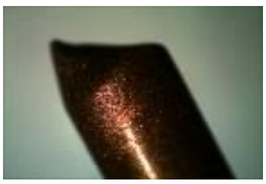
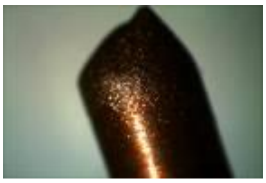
Experimental Runs	Replicate # 1	Replicate # 2	Replicate # 3
12			
$T_{on} = 4\mu s, T_{off} 6.5\ \mu s$, Duty Factor = 38%, HV = 0.7, LV = 50			
13			
$T_{on} = 6.5\mu s, T_{off} 6.5\ \mu s$, Duty Factor = 50%, HV = 0.3, LV = 30			
14			
$T_{on} = 6.5\mu s, T_{off} 6.5\ \mu s$, Duty Factor = 50%, HV = 0.7, LV = 30			
15			
$T_{on} = 6.5\mu s, T_{off} 6.5\ \mu s$, Duty Factor = 50%, HV = 0.3, LV = 50			
16			
$T_{on} = 6.5\mu s, T_{off} 6.5\ \mu s$, Duty Factor = 50%, HV = 0.7, LV = 50			

Table IV. EN 31Workpiece Surface 22 X.

4. Copper Electrode Surface which Machined EN-31. - The electrode images that machined EN-31 were taken at a magnification level of 17 X.

Experimental Runs	Replicate # 1	Replicate # 2	Replicate # 3
1			
$T_{on} = 4\mu s, T_{off} 5.5\ \mu s$, Duty Factor = 42%, HV = 0.3, LV = 30			

Experimental Runs	**Replicate # 1**	**Replicate # 2**	**Replicate # 3**
2			
T_{on} = 4µs,T_{off}5.5 µs,Duty Factor = 42%, HV = 0.7, LV = 30			
3			
T_{on} = 4µs,T_{off}5.5 µs,Duty Factor = 42%, HV = 0.3, LV = 50			
4			
T_{on} = 4µs,T_{off}5.5 µs,Duty Factor = 42%, HV = 0.7, LV = 50			
5			
T_{on} = 6.5µs,T_{off}5.5 µs,Duty Factor = 54%, HV = 0.3, LV = 30			
6			
T_{on} = 6.5µs,T_{off}5.5 µs,Duty Factor = 54%, HV = 0.7, LV = 30			
7			
T_{on} = 6.5µs,T_{off}5.5 µs,Duty Factor = 54%, HV = 0.3, LV = 50			
8			
T_{on} = 6.5µs,T_{off}5.5 µs,Duty Factor = 54%, HV = 0.7, LV = 50			

Experimental Runs	Replicate # 1	Replicate # 2	Replicate # 3
9			
$T_{on} =$ 4µs,T_{off}6.5 µs,Duty Factor = 38%, HV = 0.3, LV = 30			
10			
$T_{on} =$ 4µs,T_{off}6.5 µs,Duty Factor = 38%, HV = 0.7, LV = 30			
11			
$T_{on} =$ 4µs,T_{off}6.5 µs,Duty Factor = 38%, HV = 0.3, LV = 50			
12			
$T_{on} =$ 4µs,T_{off}6.5 µs,Duty Factor = 38%, HV = 0.7, LV = 50			
13			
$T_{on} =$ 6.5µs,T_{off}6.5 µs,Duty Factor = 50%, HV = 0.3, LV = 30			
14			
$T_{on} =$ 6.5µs,T_{off}6.5 µs,Duty Factor = 50%, HV = 0.7, LV = 30			
15			
$T_{on} =$ 6.5µs,T_{off}6.5 µs,Duty Factor = 50%, HV = 0.3, LV = 50			

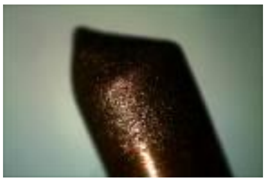
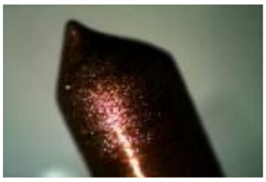
Experimental Runs	**Replicate # 1**	**Replicate # 2**	**Replicate # 3**
16			
$T_{on\,=}$ 6.5µs,T_{off}6.5 µs,Duty Factor = 50%, HV = 0.7, LV = 50			

Table V. EN 31 – Copper Electrode – 17 X.

5. Statistical Analysis of Experimental Results. -

5.1 Analysis of Results of EN-31. - The Minitab Analysis of the results of responses on EN-31 will be discussed in this section.

5.2 Analysis of Results of Machining Time (Tm) for EN-31. - The ANOVA table for Tm is prepared in Minitab considering all factors and then the significant factors are determined having p-value less than 0.05. The goal is kept in mind i.e. Minimization of Machining Time.

Factorial Fit: Tm versus HV, LV, TON, TOFF

Estimated Effects and Coefficients for Tm (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		881.3	30.82	28.59	0.000
HV	-24.2	-12.1	30.82	-0.39	0.697
LV	-155.5	-77.7	30.82	-2.52	0.017
TON	-298.6	-149.3	30.82	-4.84	0.000
TOFF	70.2	35.1	30.82	1.14	0.264
HV*LV	59.9	30.0	30.82	0.97	0.338
HV*TON	-83.0	-41.5	30.82	-1.35	0.188
HV*TOFF	3.9	2.0	30.82	0.06	0.950
LV*TON	-22.6	-11.3	30.82	-0.37	0.717
LV*TOFF	40.8	20.4	30.82	0.66	0.512
TON*TOFF	-98.9	-49.5	30.82	-1.60	0.118
HV*LV*TON	34.3	17.2	30.82	0.56	0.581
HV*LV*TOFF	-38.1	-19.0	30.82	-0.62	0.541
HV*TON*TOFF	28.0	14.0	30.82	0.45	0.653
LV*TON*TOFF	4.6	2.3	30.82	0.07	0.941
HV*LV*TON*TOFF	8.5	4.3	30.82	0.14	0.891

S = 213.560   PRESS = 3283765
R-Sq = 54.35%   R-Sq(pred) = 0.00%   R-Sq(adj) = 32.95%

Analysis of Variance for Tm (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	1426124	1426124	356531	7.82	0.000
2-Way Interactions	6	269475	269475	44912	0.98	0.452
3-Way Interactions	4	41210	41210	10302	0.23	0.922
4-Way Interactions	1	867	867	867	0.02	0.891
Residual Error	32	1459451	1459451	45608		
Pure Error	32	1459451	1459451	45608		
Total	47	3197127				

Unusual Observations for Tm

Obs	StdOrder	Tm	Fit	SE Fit	Residual	St Resid
2	2	677.00	1036.00	123.30	-359.00	-2.06R
9	9	778.00	1135.67	123.30	-357.67	-2.05R

R denotes an observation with a large standardized residual.

Table VI. ANOVA Table of Tm for EN-31 considering all factors

Estimated Coefficients for Tm using data in uncoded units

Term	Coef
Constant	1128.9
HV	-3789.8
LV	-116.122
TON	627.83
TOFF	-54.34
HV*LV	200.279
HV*TON	-296.67
HV*TOFF	907.25
LV*TON	3.6633
LV*TOFF	20.6042
TON*TOFF	-81.800
HV*LV*TON	-13.533
HV*LV*TOFF	-36.892
HV*TON*TOFF	-24.00
LV*TON*TOFF	-1.3333
HV*LV*TON*TOFF	3.4000

Table VII. Coefficient Table of Tm for EN-31 considering all factors.

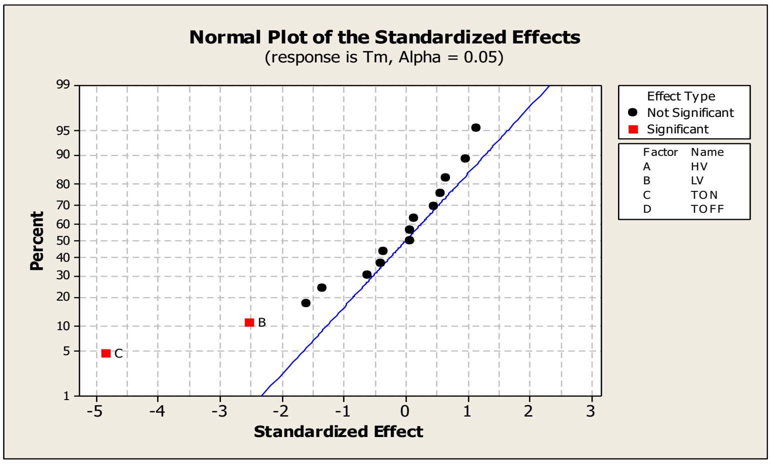


Figure II. Normal Probability Plot of the standardized effects of Tm for EN-31 considering all factors.

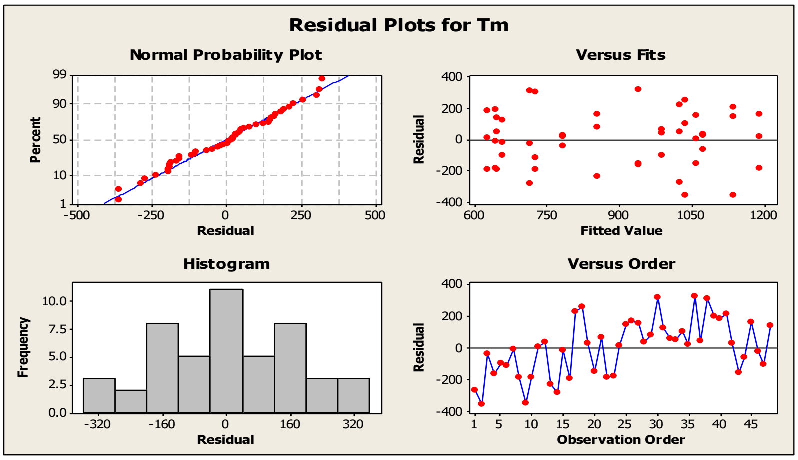


Figure III. Residual Plot of Tm for EN-31 considering all factors.

The ANOVA table as well as the Normal Probability Plot indicates that LV and Pulse On Time are significant factors when considering Tm for EN-31. Now, the model is refitted by eliminating the non-significant values and considering only LV and Pulse On Time as input factors.

Factorial Fit: Tm versus LV, TON

```
Estimated Effects and Coefficients for Tm (coded units)

TermEffectCoef  SE Coef       T       P
Constant           881.3   29.16  30.22  0.000
LV        -155.5   -77.7   29.16  -2.67  0.011
TON       -298.6  -149.3   29.16  -5.12  0.000

S = 202.053     PRESS = 2090257
R-Sq = 42.54%   R-Sq(pred) = 34.62%   R-Sq(adj) = 39.98%

Analysis of Variance for Tm (coded units)

Source          DF   Seq SS   AdjSSAdj MS      F      P
Main Effects     2  1359987  1359987  679994  16.66  0.000
Residual Error  45  1837140  1837140   40825
  Lack of Fit    1     6120     6120    6120   0.15  0.703
  Pure Error    44  1831019  1831019   41614
Total           47  3197127

Unusual Observations for Tm

ObsStdOrder      Tm      Fit  SE Fit  Residual  St Resid
  2         2  677.00  1108.37   50.51   -431.37     -2.20R

R denotes an observation with a large standardized residual.

Estimated Coefficients for Tm using data in uncoded units

Term          Coef
Constant   1819.36
LV        -7.77500
TON       -119.433
```

Table VIII. ANOVA Table of Tm for EN-31 considering significant factors.

The p-values from the ANOVA table of refitted MODEL indicate that the models and these factors are significant.

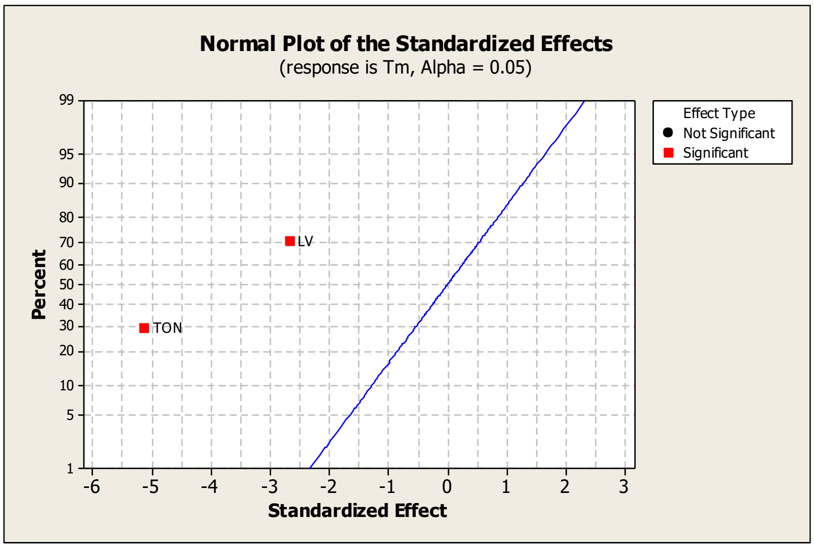


Figure IV. Normal Probability Plot of the standardized effects of Tm for EN-31 considering significant factors.

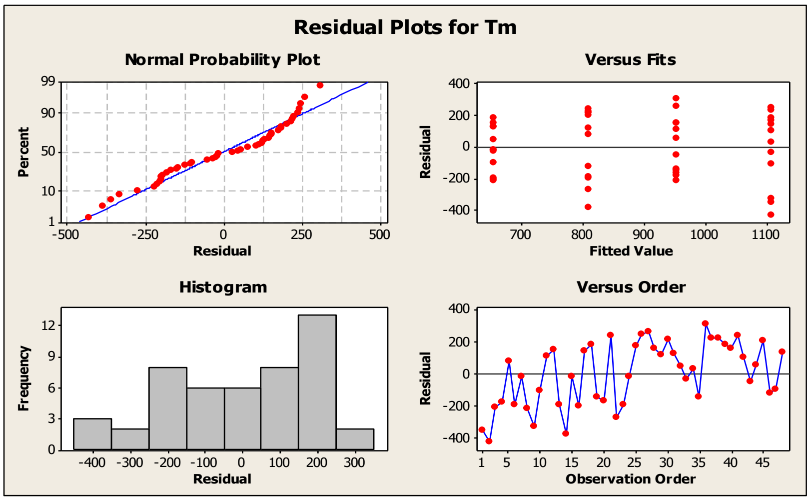


Figure V. Residual Plot of Tm for EN-31 considering all factors.

After developing the ANOVA tables for significant factors, the Main Effects Plot as well as the Interaction Plot is prepared.

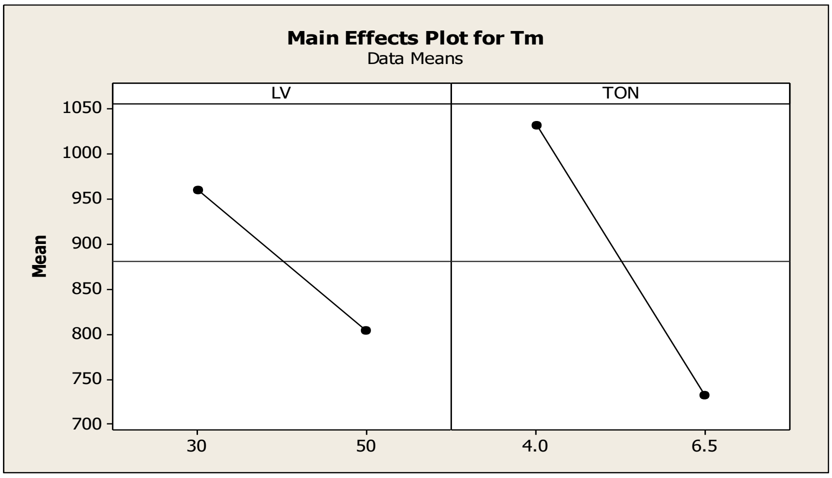


Figure VI. Main Effects Plot of Tm for EN-31 considering significant factors.

The main effects plot shows steep slope of means indicating the significance of these factors

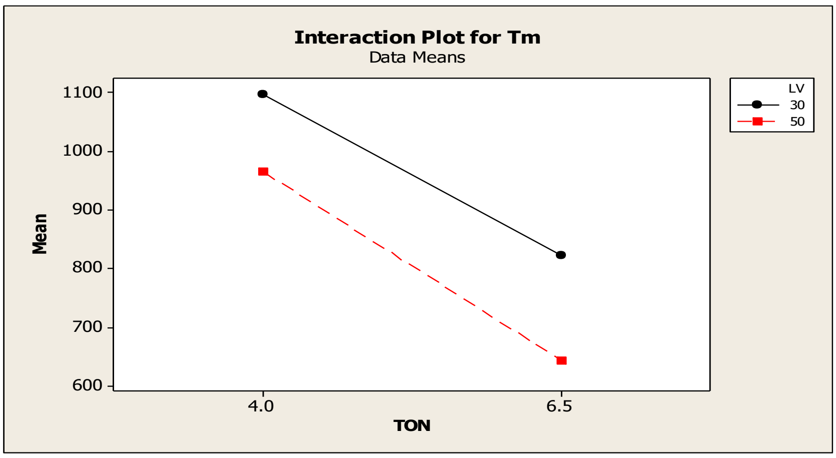


Figure VII. Residual Plot of Tm for EN-31 considering Ton & LV.

The interaction plot shows nonparallel lines of significance.

Next, the optimized values of significant factors are to be calculated. For optimization of Machining Time we have set the Target Value to '0' while the Upper Value to '427' which is the minimum value of machining time and then the value of Desirability functions is evaluated.

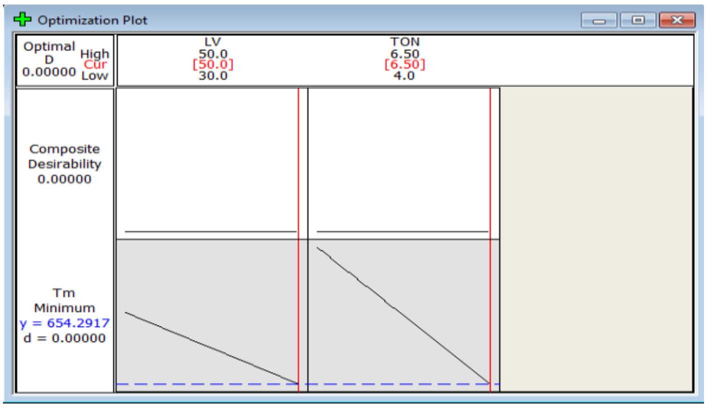


Figure VIII. Optimization Plot of Tm for EN-31 for significant factors.

d=0 emphasizes that y or response is more away from the target that is "Less emphasis on the Target", because the target was taken as "0" and response comes out to be far away from it (rather far from the Upper value that was taken as 427). It could have been d=1 or close to 1, if the target was set close to 600 and upper value was taken as greater than say 2000.

The Optimized value for LV is 50 while for TON is 6.5. For these values minimum Tm is 654.29 sec.

5.3 Analysis of Results of Material Removal Rate (MRR) for EN-31. - The ANOVA table for MRR is prepared in Minitab considering all factors and then the significant factors are determined having p-value less than 0.05. The goal is kept in mind i.e. Maximization of Material Removal Rate.

Factorial Fit: MRR versus HV, LV, TON, TOFF

Estimated Effects and Coefficients for MRR (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		0.012954	0.000587	22.06	0.000
HV	0.000994	0.000497	0.000587	0.85	0.404
LV	0.002337	0.001168	0.000587	1.99	0.055
TON	0.005532	0.002766	0.000587	4.71	0.000
TOFF	-0.001445	-0.000723	0.000587	-1.23	0.227
HV*LV	-0.001566	-0.000783	0.000587	-1.33	0.192
HV*TON	0.001478	0.000739	0.000587	1.26	0.217
HV*TOFF	-0.000456	-0.000228	0.000587	-0.39	0.700
LV*TON	0.000674	0.000337	0.000587	0.57	0.570
LV*TOFF	-0.001504	-0.000752	0.000587	-1.28	0.209
TON*TOFF	0.000801	0.000400	0.000587	0.68	0.500
HV*LV*TON	-0.000836	-0.000418	0.000587	-0.71	0.482
HV*LV*TOFF	0.000038	0.000019	0.000587	0.03	0.974
HV*TON*TOFF	-0.000484	-0.000242	0.000587	-0.41	0.683
LV*TON*TOFF	-0.000386	-0.000193	0.000587	-0.33	0.745
HV*LV*TON*TOFF	-0.000699	-0.000349	0.000587	-0.60	0.556

S = 0.00406782   PRESS = 0.00119140
R-Sq = 52.57%   R-Sq(pred) = 0.00%   R-Sq(adj) = 30.34%

Analysis of Variance for MRR (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	0.00046961	0.00046961	0.00011740	7.10	0.000
2-Way Interactions	6	0.00009845	0.00009845	0.00001641	0.99	0.448
3-Way Interactions	4	0.00001300	0.00001300	0.00000325	0.20	0.938
4-Way Interactions	1	0.00000586	0.00000586	0.00000586	0.35	0.556
Residual Error	32	0.00052951	0.00052951	0.00001655		
Pure Error	32	0.00052951	0.00052951	0.00001655		
Total	47	0.00111643				

Unusual Observations for MRR

Obs	StdOrder	MRR	Fit	SE Fit	Residual	StResid
8	8	0.026697	0.019328	0.002349	0.007369	2.22R
14	14	0.025433	0.017136	0.002349	0.008298	2.50R
23	23	0.025153	0.017658	0.002349	0.007495	2.26R
30	30	0.008880	0.017136	0.002349	-0.008255	-2.49R

R denotes an observation with a large standardized residual.

Table IX. ANOVA Table of MRR for EN-31 considering all factors

Estimated Coefficients for MRR using data in uncoded units

Term	Coef
Constant	-0.104561
HV	0.277012
LV	0.00412233
TON	0.0128062
TOFF	0.0205225
HV*LV	-0.0084327
HV*TON	-0.045838
HV*TOFF	-0.051585
LV*TON	-0.00054294
LV*TOFF	-0.00073170
TON*TOFF	-0.0027492
HV*LV*TON	0.00151000
HV*LV*TOFF	0.00148653
HV*TON*TOFF	0.0092474
LV*TON*TOFF	0.000108918
HV*LV*TON*TOFF	-2.79541E-04

Table X. Coefficient Table of MRR for EN-31 considering all factors

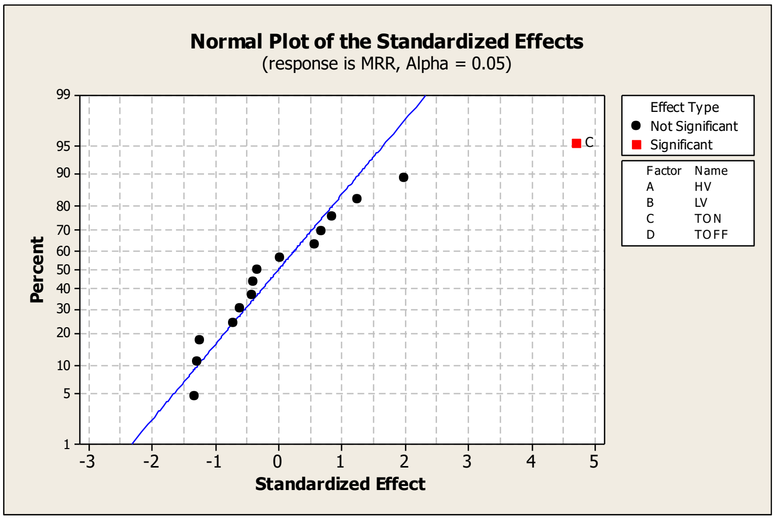


Figure IX. Normal Probability Plot of the standardized effects of MRR for EN-31 considering all factors

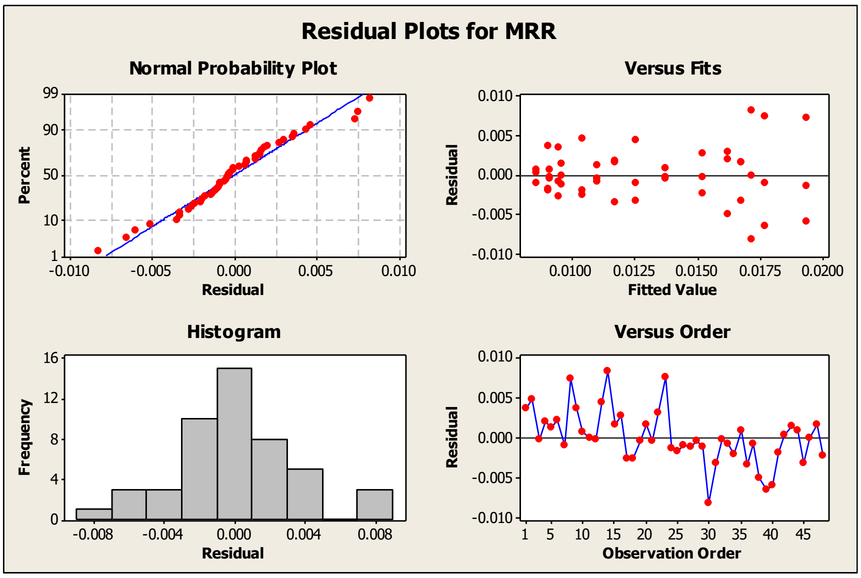


Figure X. Residual Plot of MRR for EN-31 considering all factors

The ANOVA table as well as the Normal Probability Plot indicates that Pulse On Time is a significant factor when considering MRR for EN-31. Now, the model is refitted by eliminating the non-significant values and considering only Ton as an input factor.

Factorial Fit: MRR versus TON

Estimated Effects and Coefficients for MRR (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		0.012954	0.000583	22.24	0.000
TON	0.005532	0.002766	0.000583	4.75	0.000

S = 0.00403587 PRESS = 0.000815827
R-Sq = 32.89% R-Sq(pred) = 26.93% R-Sq(adj) = 31.43%

Analysis of Variance for MRR (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	1	0.00036717	0.00036717	0.00036717	22.54	0.000
Residual Error	46	0.00074926	0.00074926	0.00001629		
Pure Error	46	0.00074926	0.00074926	0.00001629		
Total	47	0.00111643				

Unusual Observations for MRR

Obs	StdOrder	MRR	Fit	SE Fit	Residual	St Resid
8	8	0.026697	0.015720	0.000824	0.010977	2.78R
14	14	0.025433	0.015720	0.000824	0.009713	2.46R
23	23	0.025153	0.015720	0.000824	0.009433	2.39R

R denotes an observation with a large standardized residual.

Estimated Coefficients for MRR using data in uncoded units

Term	Coef
Constant	0.00133817
TON	0.00221261

Table XI. ANOVA Table of MRR for EN-31 considering significant factors

The p-values from the ANOVA table of refitted MODEL indicate that the models and these factors are significant.

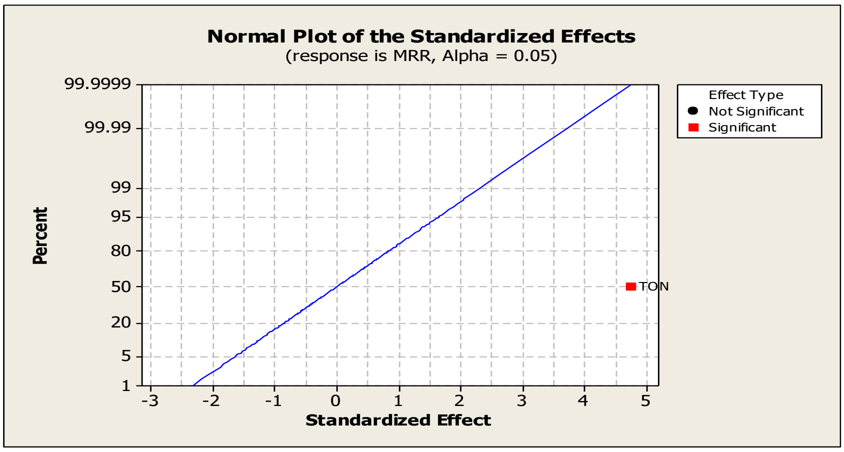


Figure XI. Normal Probability Plot of the standardized effects of MRR for EN-31 considering significant factors

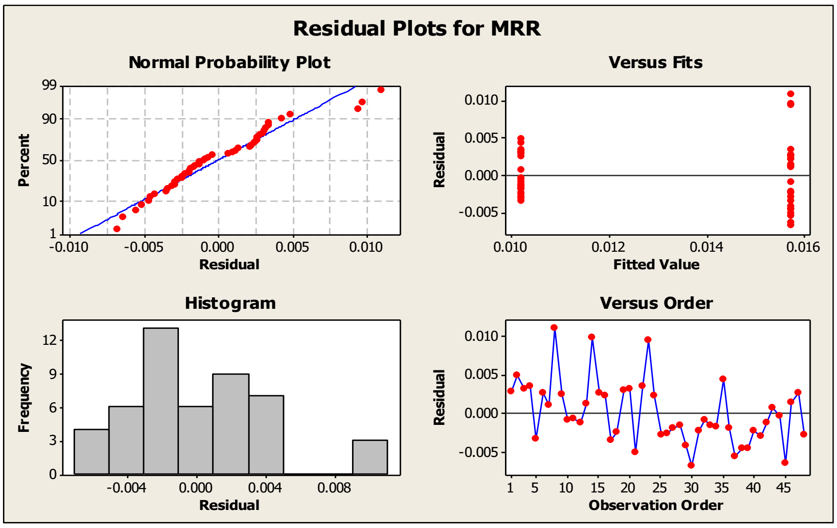


Figure XII. Residual Plot of MRR for EN-31 considering all factors.

After developing the ANOVA tables for significant factors, the Main Effects Plot as well as the Interaction Plot is prepared for MRR.

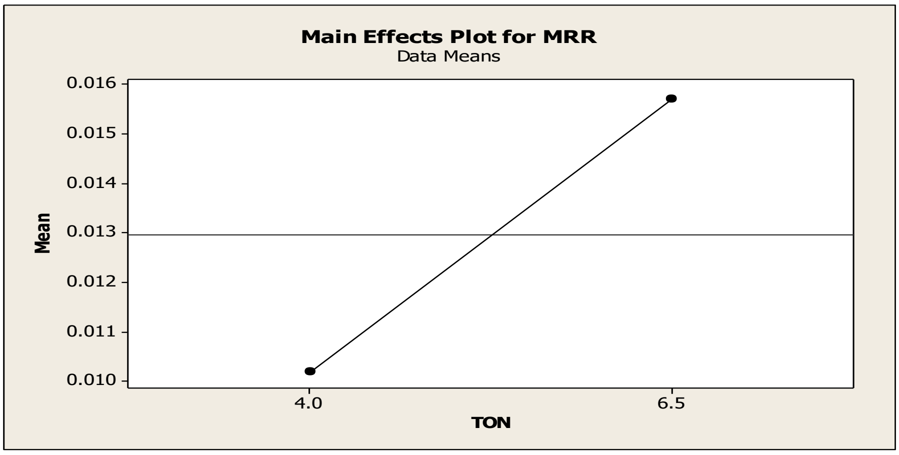


Figure XIII. Main Effects Plot of MRR for EN-31 considering significant factors.

The main effects plot shows steep slope of means indicating the significance of these factors.

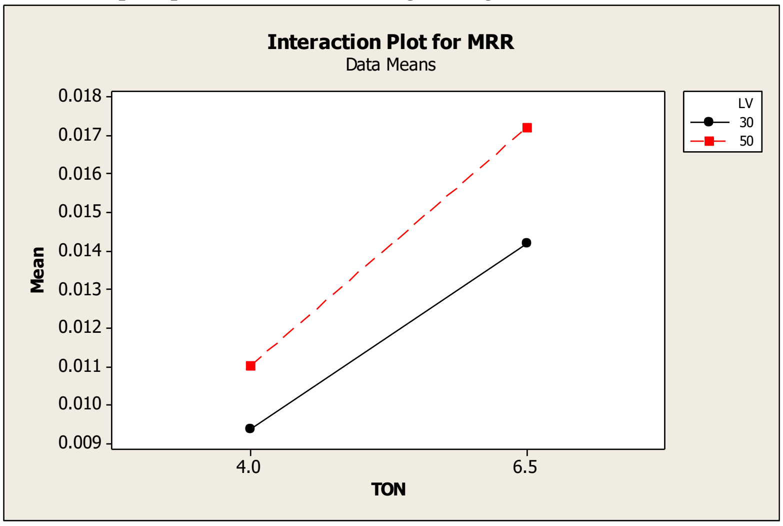


Figure XIV. Residual Plot of MRR for EN-31 considering Ton & LV.

The interaction plot is drawn between Ton and LV since LV has a p-value of 0.055 that is near significant. The interaction plot shows nonparallel lines of significance.

Next, the optimized values of significant factors are to be calculated. For optimization of Material Removal Rate, we have set the Target Value to '1' while the Lower Value to '.0267' which is the Maximum value of Material Removal Rate and then the value of Desirability functions is evaluated.

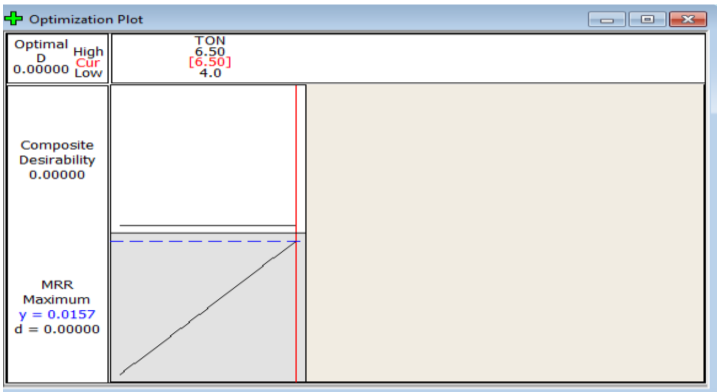


Figure XV. Optimization Plot of MRR for EN-31 for significant factors

The Weight of Specific Desirability Function (d) nearly equals to 0 i.e. emphasis on the Target (0). The Desirability for a Response increases linearly.

d=0 emphasizes that y or response is more away from the target that is "Less emphasis on the Target", because the target was taken as "1" and response comes out to be far away from it (rather far from the Lower value that was taken as 0.02670 g/min). It could have been d=1 or close to 1, if the target was set close to 0.015 and lower value was taken as less than say 0.014.

The Optimized value for Ton is 6.5 µs for which maximum MRR is 0.0157 g/min.

5.5 Analysis of Results of Electrode Wear Rate (EW) for EN-31. - The ANOVA table for EW is prepared in Minitab considering all factors and then the significant factors are determined having p-value less than 0.05. The goal is kept in mind i.e. Minimization of Electrode Wear Rate.

Factorial Fit: EW versus HV, LV, TON, TOFF

```
Estimated Effects and Coefficients for EW (coded units)

TermEffectCoef   SE Coef      T      P
Constant                    0.009496  0.000361  26.32  0.000
HV                0.000313   0.000156  0.000361   0.43  0.668
LV                0.001913   0.000956  0.000361   2.65  0.012
TON               0.003077   0.001538  0.000361   4.26  0.000
TOFF             -0.000209  -0.000104  0.000361  -0.29  0.774
HV*LV            -0.000790  -0.000395  0.000361  -1.10  0.282
HV*TON            0.001016   0.000508  0.000361   1.41  0.169
HV*TOFF          -0.000106  -0.000053  0.000361  -0.15  0.884
LV*TON            0.000601   0.000300  0.000361   0.83  0.411
LV*TOFF          -0.000712  -0.000356  0.000361  -0.99  0.331
TON*TOFF          0.000931   0.000465  0.000361   1.29  0.206
HV*LV*TON        -0.000508  -0.000254  0.000361  -0.70  0.487
HV*LV*TOFF        0.000098   0.000049  0.000361   0.14  0.893
HV*TON*TOFF       0.000088   0.000044  0.000361   0.12  0.903
LV*TON*TOFF      -0.000314  -0.000157  0.000361  -0.44  0.666
HV*LV*TON*TOFF   -0.000291  -0.000146  0.000361  -0.40  0.689

S = 0.00249984    PRESS = 0.000449941
R-Sq = 50.68%    R-Sq(pred) = 0.00%    R-Sq(adj) = 27.57%

Analysis of Variance for EW (coded units)

Source               DF      Seq SS      Adj SS      Adj MS     F      P
Main Effects          4  0.00015918  0.00015918  0.00003980  6.37  0.001
2-Way Interactions    6  0.00004083  0.00004083  0.00000681  1.09  0.390
3-Way Interactions    4  0.00000448  0.00000448  0.00000112  0.18  0.947
4-Way Interactions    1  0.00000102  0.00000102  0.00000102  0.16  0.689
Residual Error       32  0.00019997  0.00019997  0.00000625
  Pure Error         32  0.00019997  0.00019997  0.00000625
Total                47  0.00040549

Unusual Observations for EW

ObsStdOrder        EW       Fit    SE Fit   Residual   StResid
 14        14  0.017283  0.012053  0.001443   0.005230      2.56R
 23        23  0.016638  0.012322  0.001443   0.004316      2.11R
 30        30  0.007887  0.012053  0.001443  -0.004166     -2.04R

R denotes an observation with a large standardized residual.
```

Table XII. ANOVA Table of EW for EN-31 considering all factors

```
Estimated Coefficients for EW using data in uncoded units

Term                    Coef
Constant           -0.031162
HV                  0.149405
LV                0.00141753
TON                0.0017536
TOFF               0.0078448
HV*LV            -0.00362478
HV*TON            -0.0239692
HV*TOFF           -0.0287848
LV*TON          -1.23684E-04
LV*TOFF         -2.69289E-04
TON*TOFF         -0.00075509
HV*LV*TON         0.00059704
HV*LV*TOFF        0.00066003
HV*TON*TOFF        0.0050104
LV*TON*TOFF      0.000033078
HV*LV*TON*TOFF  -1.16425E-04
```

Table XIII. Coefficient Table of EW for EN-31 considering all factors

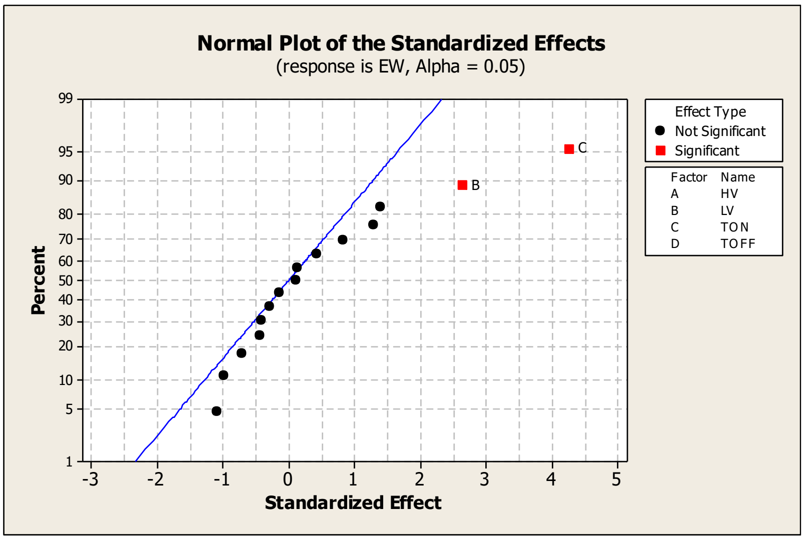


Figure XVI. Normal Probability Plot of the standardized effects of EW for EN-31 considering all factors

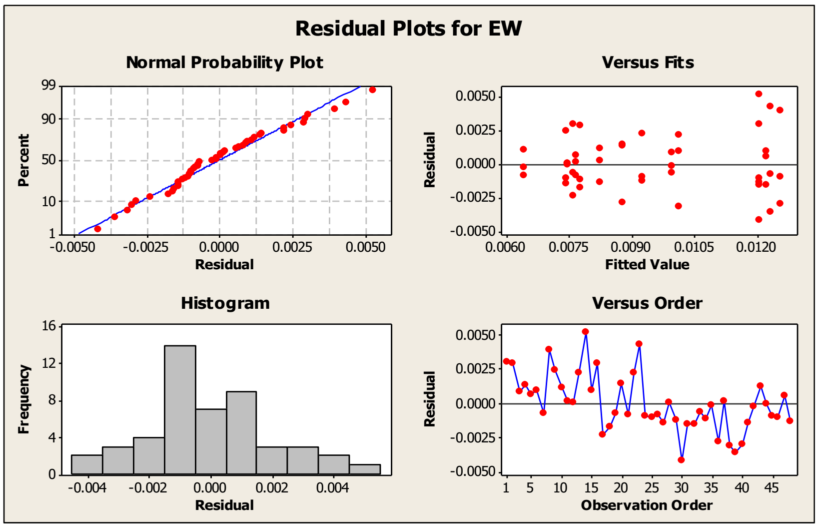


Figure XVII. Residual Plot of EW for EN-31 considering all factors

The ANOVA table as well as the Normal Probability Plot indicates that LV and Ton are significant factors when considering EW for EN-31. Now, the model is refitted by eliminating the non-significant values and considering only LV and Ton as input factors.

Factorial Fit: EW versus LV, TON

```
Estimated Effects and Coefficients for EW (coded units)

TermEffectCoef   SE Coef      T      P
Constant           0.009496  0.000339  28.02  0.000
LV        0.001913  0.000956  0.000339   2.82  0.007
TON       0.003077  0.001538  0.000339   4.54  0.000

S = 0.00234761   PRESS = 0.000282177
R-Sq = 38.84%    R-Sq(pred) = 30.41%   R-Sq(adj) = 36.12%

Analysis of Variance for EW (coded units)

Source          DF      Seq SS      Adj SS      Adj MS      F      P
Main Effects     2  0.00015748  0.00015748  0.00007874  14.29  0.000
Residual Error  45  0.00024801  0.00024801  0.00000551
  Lack of Fit    1  0.00000433  0.00000433  0.00000433   0.78  0.381
  Pure Error    44  0.00024368  0.00024368  0.00000554
Total           47  0.00040549

Unusual Observations for EW

ObsStdOrder         EW       Fit    SE Fit  Residual  St Resid
 14        14  0.017283  0.010078  0.000587  0.007206      3.17R
 23        23  0.016638  0.011990  0.000587  0.004647      2.04R

R denotes an observation with a large standardized residual.

Estimated Coefficients for EW using data in uncoded units

Term            Coef
Constant  -0.00079057
LV        9.56321E-05
TON        0.00123065
```

Table XIV. ANOVA Table of EW for EN-31 considering significant factors

The p-values from the ANOVA table of the refitted MODEL indicate that the models as well as these factors are significant.

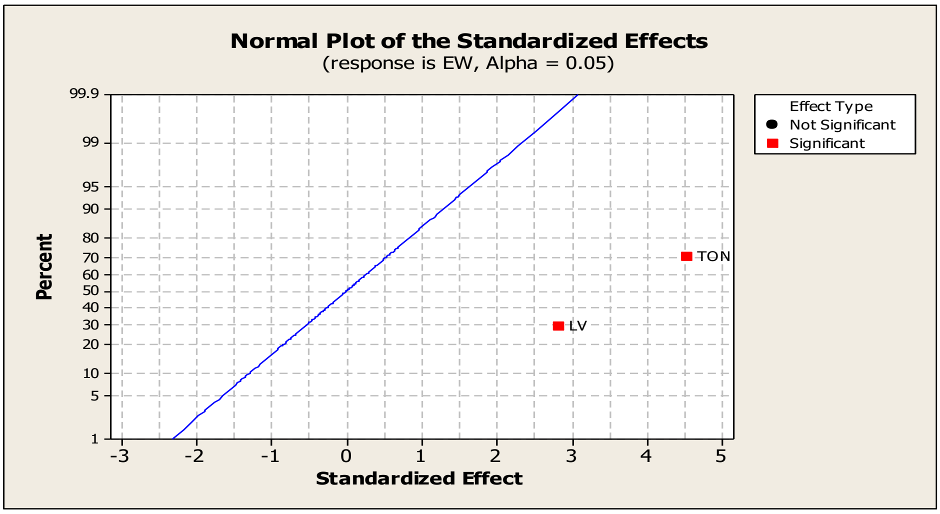


Figure XVIII. Normal Probability Plot of the standardized effects of EW for EN-31 considering significant factors

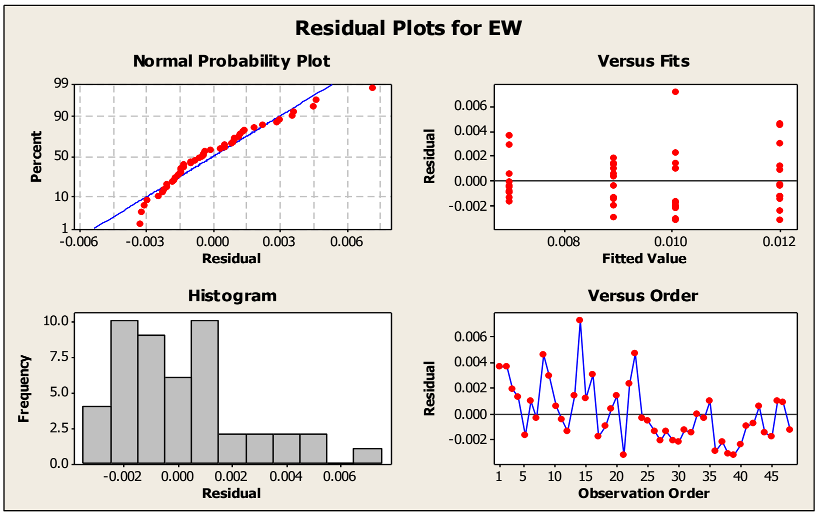


Figure XIX. Residual Plot of EW for EN-31 considering all factors

After developing ANOVA tables for significant factors, Main Effects Plot and Interaction Plot is prepared for EW.

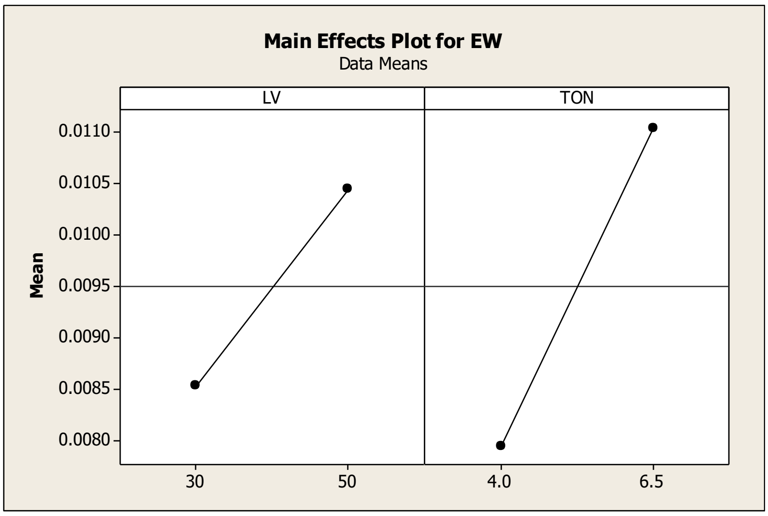


Figure XX. Main Effects Plot of EW for EN-31 considering significant factors

The main effects plot shows steep slope of means indicating the significance of these factors.

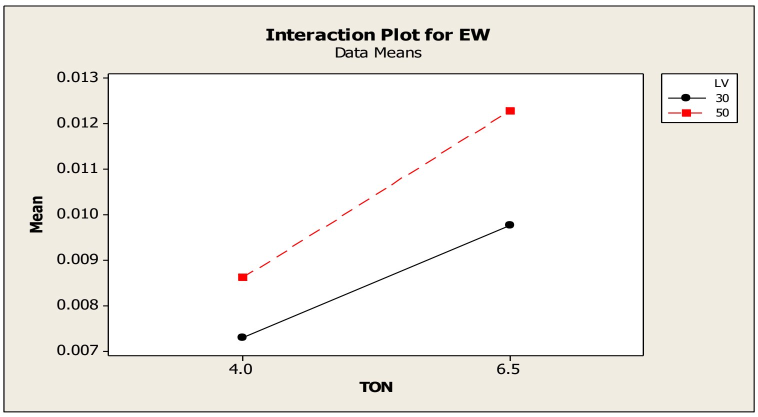


Figure XXI. Residual Plot of EW for EN-31 considering Ton & LV

The interaction plot shows nonparallel lines of significance.

Next, the optimized values of significant factors are to be calculated. For optimization of Electrode Wear Rate we have set the Target Value to '0' while the Upper Value to '0.00523 g/min' which is the Minimum value of Electrode Wear Rate and then the value of Desirability functions is evaluated.

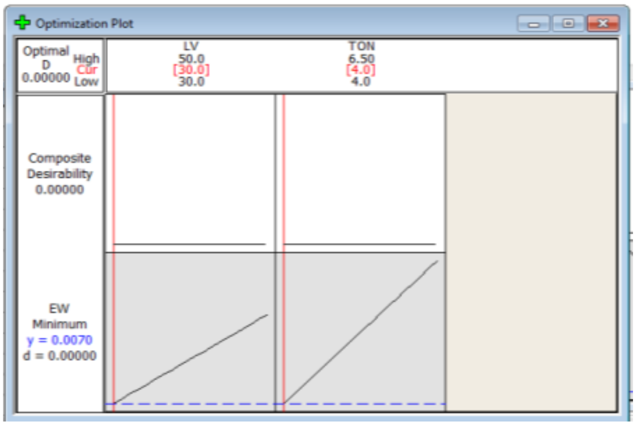


Figure XXII. Optimization Plot of EW for EN-31 for significant factors

The Weight of Specific Desirability Function (d) nearly equals to 0 i.e. emphasis on the Target (0). The Desirability for a Response increases linearly.

d=0 emphasizes that y or response is more away from the target that is "Less emphasis on the Target", because the target was taken as "1" and response comes out to be far away from it (rather far from the Upper value that was taken as 0.00523 g/min). It could have been d=1 or close to 1, if the target was set close to 0.015 and lower value was taken as less than say 0.014.

The Optimized value for LV is 30 & Ton is4 µs for whichminimumEW is 0.07 g/min.

5.6 Analysis of Results of Surface Roughness (Ra) for EN-31. - The ANOVA table for Ra is prepared in Minitab considering all factors and then the significant factors are determined having p-value less than 0.05. The goal is kept in mind i.e. Minimization of Surface Roughness.

Factorial Fit: Ra versus HV, LV, TON, TOFF

Estimated Effects and Coefficients for Ra (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		0.027354	0.002346	11.66	0.000
HV	0.000542	0.000271	0.002346	0.12	0.909
LV	-0.003125	-0.001562	0.002346	-0.67	0.510
TON	-0.002125	-0.001062	0.002346	-0.45	0.654
TOFF	-0.003792	-0.001896	0.002346	-0.81	0.425
HV*LV	0.000375	0.000188	0.002346	0.08	0.937
HV*TON	-0.000625	-0.000312	0.002346	-0.13	0.895
HV*TOFF	-0.006958	-0.003479	0.002346	-1.48	0.148
LV*TON	-0.001625	-0.000813	0.002346	-0.35	0.731
LV*TOFF	-0.001958	-0.000979	0.002346	-0.42	0.679
TON*TOFF	0.001542	0.000771	0.002346	0.33	0.745
HV*LV*TON	0.006875	0.003437	0.002346	1.47	0.153
HV*LV*TOFF	0.002208	0.001104	0.002346	0.47	0.641
HV*TON*TOFF	0.003375	0.001688	0.002346	0.72	0.477
LV*TON*TOFF	0.001042	0.000521	0.002346	0.22	0.826
HV*LV*TON*TOFF	-0.002458	-0.001229	0.002346	-0.52	0.604

S = 0.0162519   PRESS = 0.019017
R-Sq = 18.27%   R-Sq(pred) = 0.00%   R-Sq(adj) = 0.00%

Analysis of Variance for Ra (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	0.0003474	0.00034742	0.00008685	0.33	0.857
2-Way Interactions	6	0.0006936	0.00069362	0.00011560	0.44	0.848
3-Way Interactions	4	0.0007754	0.00077542	0.00019385	0.73	0.576
4-Way Interactions	1	0.0000725	0.00007252	0.00007252	0.27	0.604
Residual Error	32	0.0084520	0.00845200	0.00026412		
Pure Error	32	0.0084520	0.00845200	0.00026412		
Total	47	0.0103410				

Unusual Observations for Ra

Obs	StdOrder	Ra	Fit	SE Fit	Residual	StResid
5	5	0.071000	0.031333	0.009383	0.039667	2.99R
18	18	0.015000	0.041667	0.009383	-0.026667	-2.01R
25	25	0.059000	0.031667	0.009383	0.027333	2.06R
34	34	0.074000	0.041667	0.009383	0.032333	2.44R

R denotes an observation with a large standardized residual.

Table XV. ANOVA Table of Ra for EN-31 considering all factors.

Estimated Coefficients for Ra using data in uncoded units

Term	Coef
Constant	-1.28144
HV	2.43092
LV	0.0263475
TON	0.200975
TOFF	0.193233
HV*LV	-0.0447250
HV*TON	-0.373250
HV*TOFF	-0.356333
LV*TON	-0.00420250
LV*TOFF	-0.00376667
TON*TOFF	-0.0285167
HV*LV*TON	0.0072750
HV*LV*TOFF	0.0062667
HV*TON*TOFF	0.0528333
LV*TON*TOFF	0.00057500
HV*LV*TON*TOFF	-0.00098333

Table XVI. Coefficient Table of Ra for EN-31 considering all factors

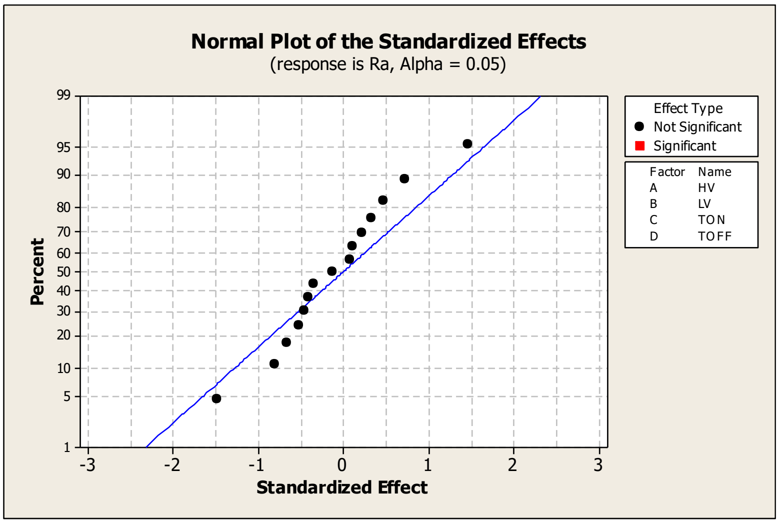


Figure XXIII. Normal Probability Plot of the standardized effects of Ra for EN-31 considering all factors

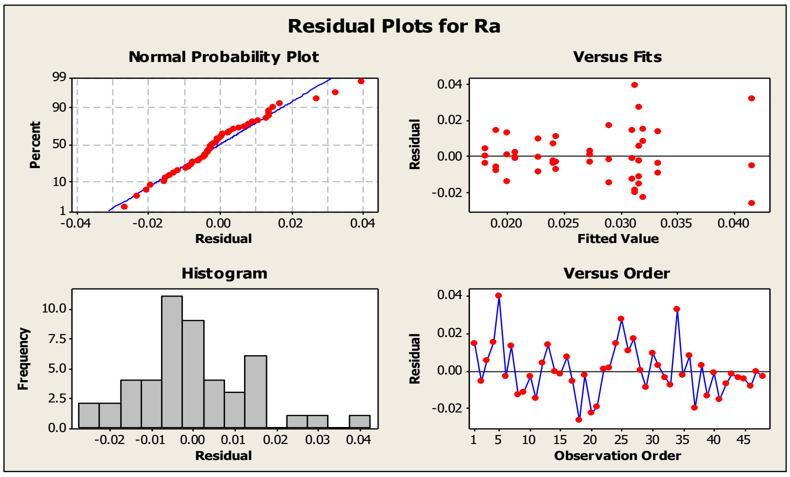


Figure XIV. Residual Plot of Ra for EN-31 considering all factors.

The ANOVA table as well as the Normal Probability Plot indicates there is no significant factor when considering Ra for EN-31. So, no need to refit the model. Now, the Main Effects Plot as well as the Interaction Plot is prepared for Ra.

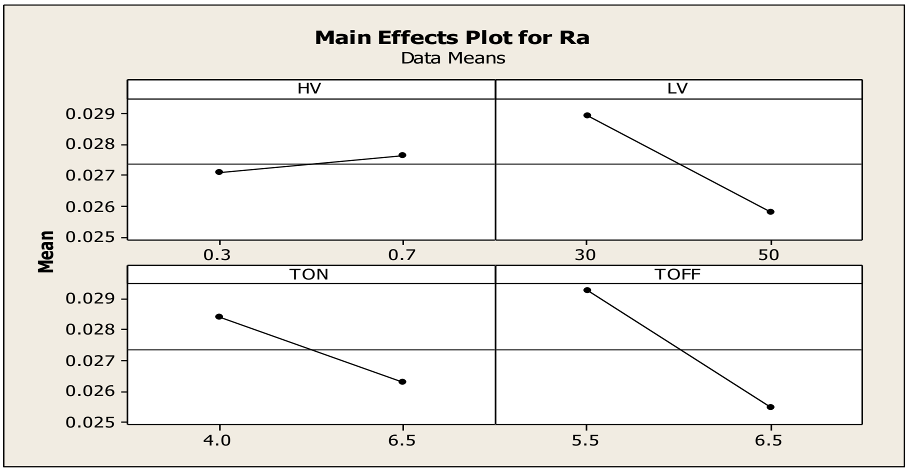


Figure XXV. Main Effects Plot of Ra for EN-31 considering all factors.

The main effects plot shows steep slope of means indicating the significance of these factors.

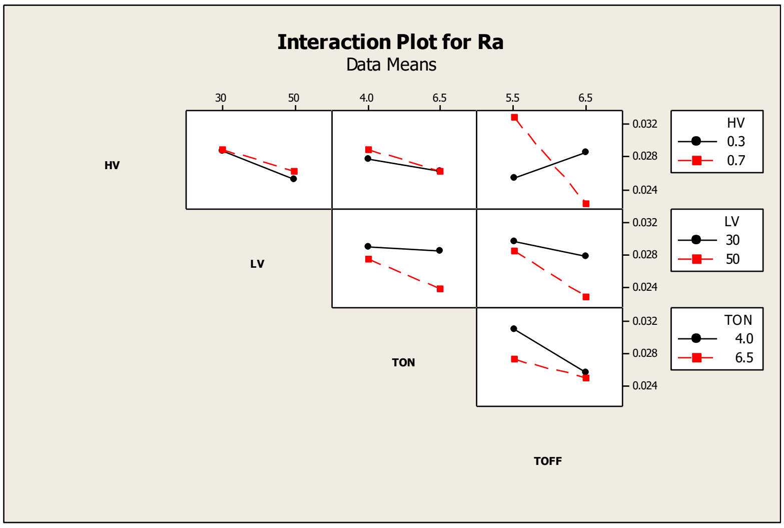


Figure XXVI. Residual Plot of Ra for EN-31 for all factors.

The interaction plot shows nonparallel lines of significance.

Next, the optimized values of significant factors are to be calculated. For optimization of Surface Roughness, we have set the Target Value to ‘0’ while the Upper Value to ‘0.006 mm’ which is the Minimum value of Surface Roughness and then the value of Desirability functions is evaluated.

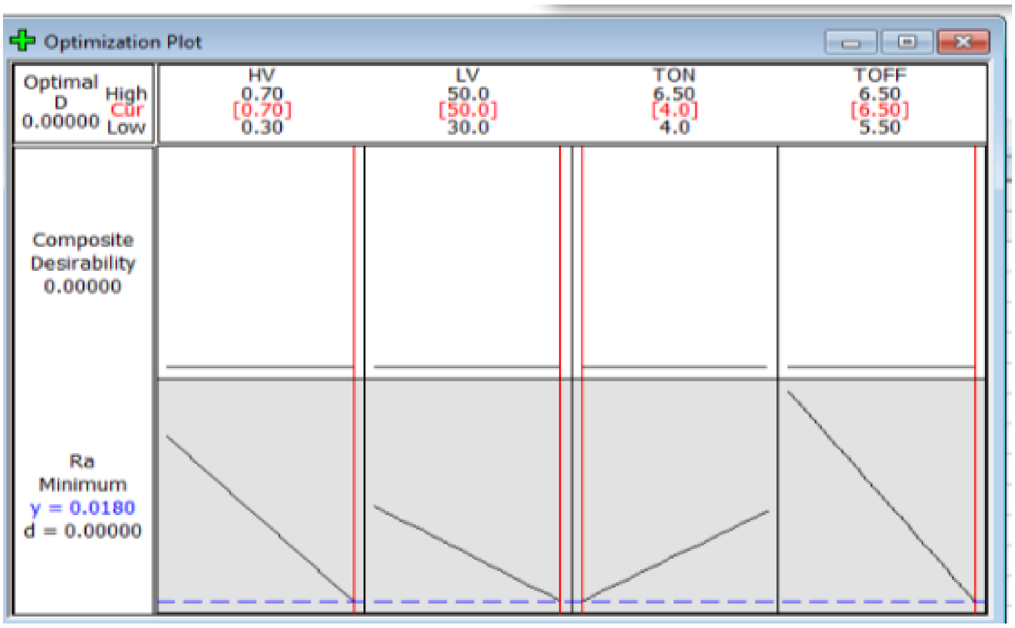


Figure XXVII. Optimization Plot of Ra for EN-31 for significant factors.

d=0 emphasizes that y or response is more away from the target that is “Less emphasis on the Target”, because the target was taken as “0” and response comes out to be far away from it (rather far from the Upper value that was taken as 0.006 mm). It could have been d=1 or close to 1, if the target was set close to 0.009 and upper value was taken as greater than say 0.015.

The Optimized value for LV is 50, HV is 0.7,Ton is 4.0 µs while Toff is 6.5 µs. For these values minimum Ra is 0.018mm.

5.7 Analysis of Results of Base Radius (R) for EN-31. - The ANOVA table for R is prepared in Minitab considering all factors and then the significant factors are determined having p-value less than 0.05. The goal is kept in mind i.e. Optimization of Base Radius to the dimensions of electrode.

Factorial Fit: R versus HV, LV, TON, TOFF

```
Estimated Effects and Coefficients for R (coded units)

TermEffectCoef  SE Coef       T      P
Constant                 1.58946  0.01629  97.57  0.000
HV             -0.04275  -0.02137  0.01629  -1.31  0.199
LV             -0.03417  -0.01708  0.01629  -1.05  0.302
TON            -0.02550  -0.01275  0.01629  -0.78  0.440
TOFF           -0.01233  -0.00617  0.01629  -0.38  0.708
HV*LV          -0.01850  -0.00925  0.01629  -0.57  0.574
HV*TON         -0.00100  -0.00050  0.01629  -0.03  0.976
HV*TOFF         0.01433   0.00717  0.01629   0.44  0.663
LV*TON          0.00492   0.00246  0.01629   0.15  0.881
LV*TOFF        -0.00892  -0.00446  0.01629  -0.27  0.786
TON*TOFF       -0.00575  -0.00287  0.01629  -0.18  0.861
HV*LV*TON      -0.07525  -0.03762  0.01629  -2.31  0.028
HV*LV*TOFF     -0.03958  -0.01979  0.01629  -1.21  0.233
HV*TON*TOFF    -0.03492  -0.01746  0.01629  -1.07  0.292
LV*TON*TOFF    -0.02383  -0.01192  0.01629  -0.73  0.470
HV*LV*TON*TOFF -0.06767  -0.03383  0.01629  -2.08  0.046

S = 0.112858    PRESS = 0.917065
R-Sq = 34.74%   R-Sq(pred) = 0.00%   R-Sq(adj) = 4.14%

Analysis of Variance for R (coded units)

Source              DF    Seq SS    Adj SS    Adj MS     F      P
Main Effects         4  0.045567  0.045567  0.011392  0.89  0.479
2-Way Interactions   6  0.008225  0.008225  0.001371  0.11  0.995
3-Way Interactions   4  0.108199  0.108199  0.027050  2.12  0.101
4-Way Interactions   1  0.054945  0.054945  0.054945  4.31  0.046
Residual Error      32  0.407585  0.407585  0.012737
  Pure Error        32  0.407585  0.407585  0.012737
Total               47  0.624522

Unusual Observations for R

ObsStdOrder        R      Fit   SE Fit  Residual  St Resid
 46        46  1.86000  1.66700  0.06516   0.19300      2.09R

R denotes an observation with a large standardized residual.
```

Table XVII. ANOVA Table of R for EN-31 considering all factors.

Estimated Coefficients for R using data in uncoded units

Term	Coef
Constant	-6.85617
HV	21.4526
LV	0.272280
TON	2.08093
TOFF	1.69079
HV*LV	-0.659463
HV*TON	-5.05800
HV*TOFF	-4.08742
LV*TON	-0.0620383
LV*TOFF	-0.0520358
TON*TOFF	-0.399833
HV*LV*TON	0.147350
HV*LV*TOFF	0.122308
HV*TON*TOFF	0.943000
LV*TON*TOFF	0.0116267
HV*LV*TON*TOFF	-0.0270667

Table XVIII. Coefficient Table of R for EN-31 considering all factors

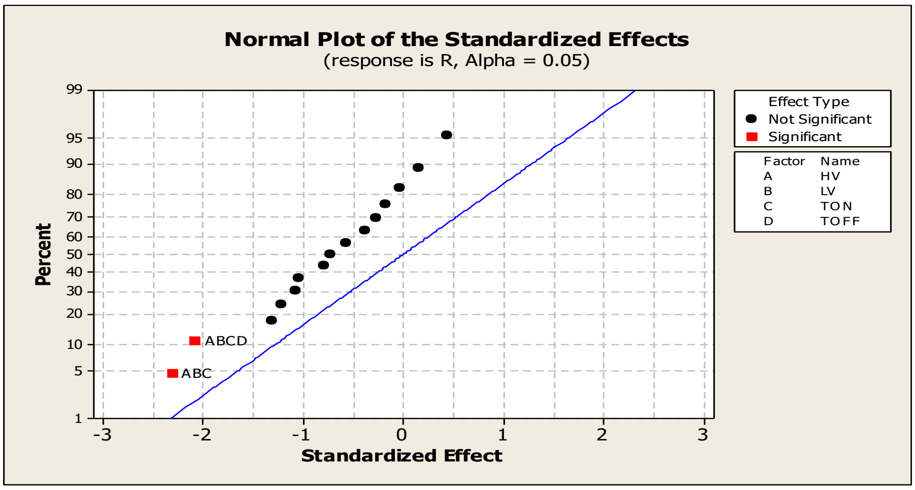


Figure XXVIII. Normal Probability Plot of the standardized effects of R for EN-31 considering all factors.

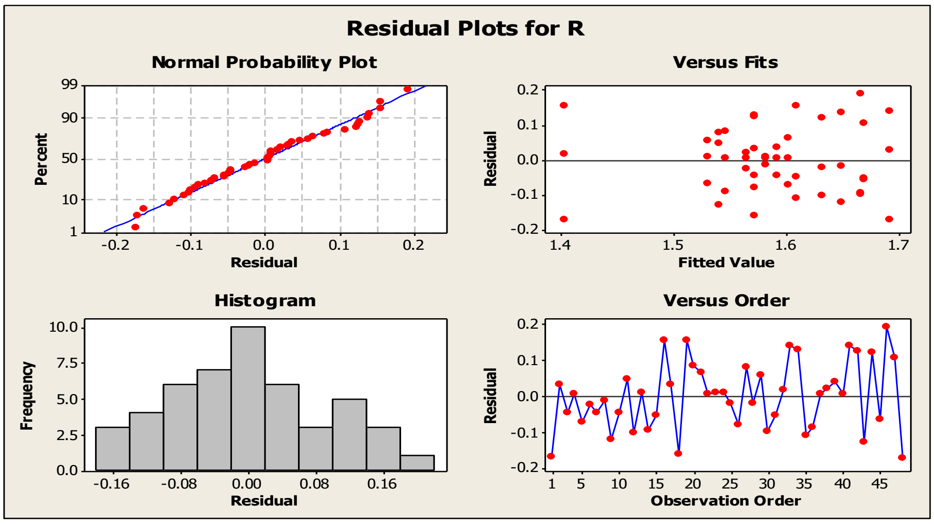


Figure XXIX. Residual Plot of R for EN-31 considering all factors.

The ANOVA table as well as the Normal Probability Plot indicates that the interactions ***HV*LV*TON*** and ***HV*HV*LV*TON*** are significant factors when considering R for EN-31. Now, the model is refitted by eliminating the non-significant values and considering only *HV*LV*TON* and *HV*HV*LV*TON* as input factors.

Factorial Fit: R versus HV, LV, TON, TOFF

Estimated Effects and Coefficients for R (coded units)

TermEffectCoef		SE Coef	T	P	
Constant		1.58946	0.01522	104.41	0.000
HV	-0.04275	-0.02137	0.01522	-1.40	0.168
LV	-0.03417	-0.01708	0.01522	-1.12	0.268
TON	-0.02550	-0.01275	0.01522	-0.84	0.407
TOFF	-0.01233	-0.00617	0.01522	-0.41	0.688
HV*LV*TON	-0.07525	-0.03762	0.01522	-2.47	0.018
HV*LV*TON*TOFF	-0.06767	-0.03383	0.01522	-2.22	0.032

S = 0.105467   PRESS = 0.625079
R-Sq = 26.97%   R-Sq(pred) = 0.00%   R-Sq(adj) = 16.29%

Analysis of Variance for R (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	0.04557	0.04557	0.011392	1.02	0.406
3-Way Interactions	1	0.06795	0.06795	0.067951	6.11	0.018
4-Way Interactions	1	0.05495	0.05495	0.054945	4.94	0.032
Residual Error	41	0.45606	0.45606	0.011123		
Lack of Fit	9	0.04847	0.04847	0.005386	0.42	0.913
Pure Error	32	0.40758	0.40758	0.012737		
Total	47	0.62452				

Unusual Observations for R

ObsStdOrder		R	Fit	SE Fit	Residual	St Resid
46	46	1.86000	1.63771	0.04028	0.22229	2.28R
48	48	1.23000	1.46063	0.04028	-0.23063	-2.37R

R denotes an observation with a large standardized residual.

* NOTE * Estimated regression coefficients in uncoded units are not available because the model is non-hierarchical.

Table XIX. ANOVA Table of R for EN-31 considering significant factors

The p-values from the ANOVA table of refitted MODEL indicate that the models and these factors are significant.

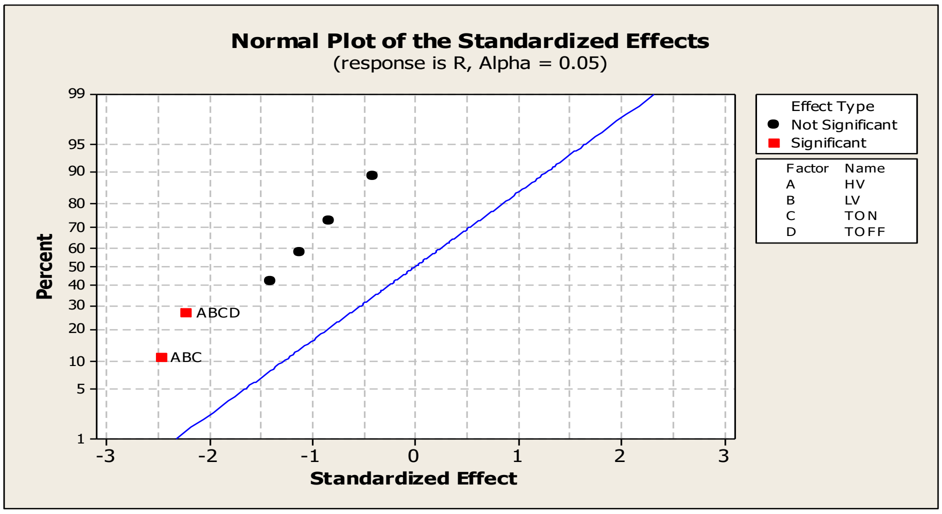


Figure XXX. Normal Probability Plot of the standardized effects of R for EN-31 considering significant factors.

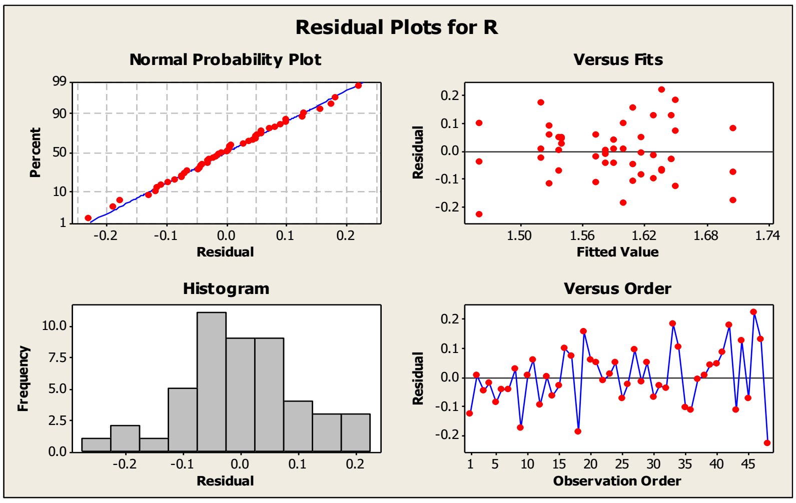


Figure XXXI. Residual Plot of R for EN31 considering significant factors.

After developing the ANOVA tables for significant factors Main Effects Plot and Interaction Plot is prepared for R.

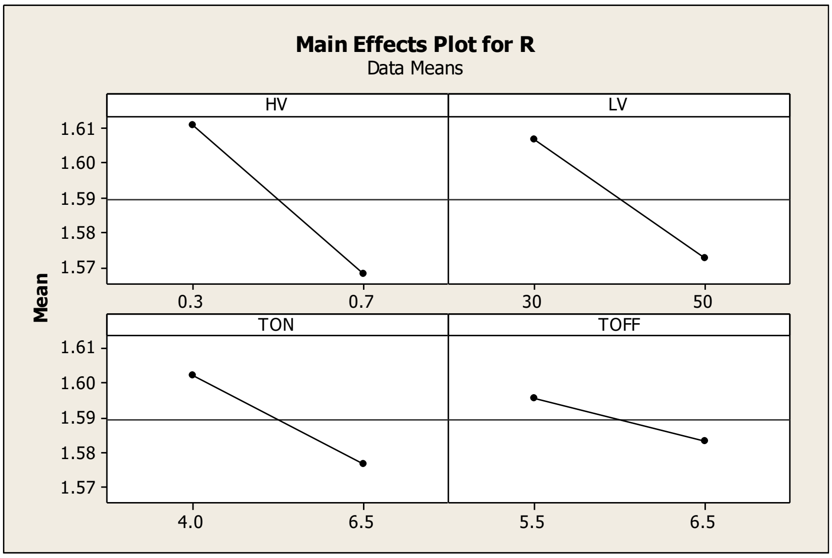


Figure XXXII. Main Effects Plot of R for EN-31 considering significant factors

The main effects plot shows steep slope of means indicating the significance of these factors.

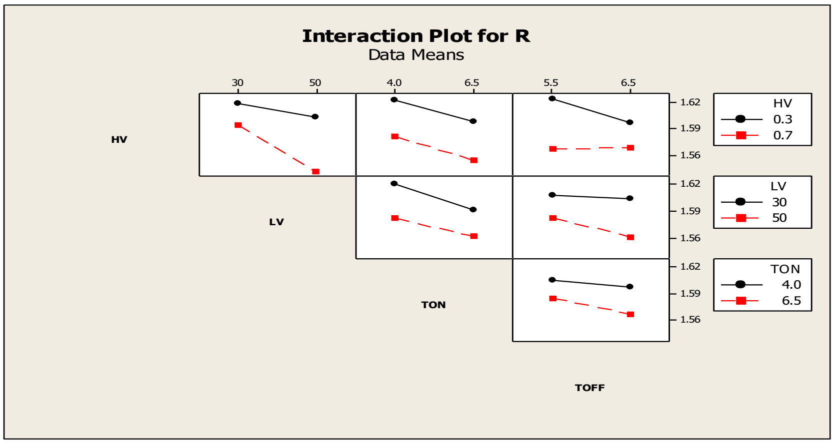


Figure XXXIII. Residual Plot of R for EN-31 considering significant factors

The interaction plot shows non parallel lines of significance.

Next, the optimized values of significant factors are to be calculated. For optimization of Base Radius value, we have set the Target Value to '1.5' while the Upper & Lower Values to 1.55 &1.45respectively and then the value of Desirability function is evaluated.

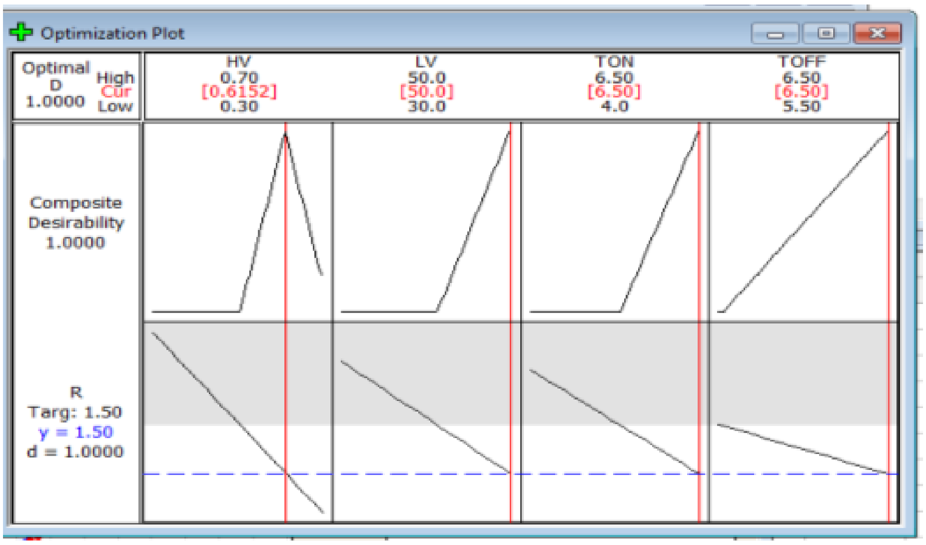


Figure XXXIV. Optimization Plot of R for EN-31 for significant factors

The Weight of Specific Desirability Function (d) nearly to 1 i.e. emphasis on the Target. The Desirability for a Response increases linearly.

The Optimized value for HV is 0.6152, LV is 50, Ton is 6.5 and Toff is also 6.5. For these values optimized R is 1.5 mm.